(Twisted) toroidal compactification of pp waves

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The maximally supersymmetric type IIB pp wave is compactified on spatial circles, with and without an auxiliary rotational twist. All spatial circles of constant radius are identified. Without the twist, an S^1 compactification can preserve 24, 20 or 16 supercharges. T^2 compactifications can preserve 20, 18 or 16 supercharges; T^3 compactifications can preserve 18 or 16 supercharges and higher compactifications preserve 16 supercharges. The world sheet theory of this background is discussed. The T dual and decompactified type IIA and M-theoretic solutions which preserve 24 supercharges are given. Some comments are made regarding the AdS parent and the CFT description.

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I. INTRODUCTION

In addition to the maximally symmetric $\mathrm{AdS}_5\times S^5$ and Minkowski backgrounds, it has recently been realized that there is an additional background for the type IIB string which preserves 32 supercharges, namely the pp wave [1]. As this background is achieved as a Penrose limit of $\mathrm{AdS}_5\times S^5$ [2,3], string theory on this background has a conformal field theory (CFT) dual [4]. Remarkably, this CFT is powerful enough to see the perturbative string spectrum. These observations have caused a flurry of interest [5–19].

Moreover, the pp wave has 30 isometries—the same number as $\mathrm{AdS}_5\times S^5$ by virtue of the Penrose limit—of which sixteen are spatial and noncompact. These therefore provide a way to compactify the pp wave to lower dimensions. In fact, the maximal number of commuting, noncompact and spatial Killing vectors is eight, the same number as for toroidal compactification of ordinary Minkowski space to the light cone.

The study of toroidal compactifications has provided a very rich structure of phenomena and dualities. In this paper such a study is initiated for pp waves. The first step, given after a review of the pp-wave geometry in Sec. II, is to identify spacelike isometries on which to compactify. This is done in Sec. III.

An analysis of Killing spinors, in Sec. III, shows that compactification preserves at least half the supersymmetries, up to 3/4, or 24 supercharges. While surprising, this can occur because of the reduced rotational symmetry of the system, due to the nontrivial curvatures. In [20] an analysis of the central charge matrix gave rise to the possibility of 3/4 Bogomol'nyl-Prasad-Sommerfield (BPS) states; it would be interesting if the geometry studied here is employing this mechanism.

In Sec. VI, the T-dual solution is given and lifted to M theory. It is then shown that the M-theory solution also preserves 24 supercharges. This is because the Killing spinors on the type IIB side do not involve the compact coordinate. Otherwise there would be "supersymmetry without supersymmetry" [21,22].

Since the isometries derive from $\mathrm{AdS}_5\times S^5$ isometries, it is possible to find the corresponding quotient of AdS_5, which has a CFT dual. As discussed in Sec. VII, the quotient turns out to be one of those previously discussed by Ghosh and Mukhi [23].

In addition we can perform a Melvin-like twist, as was previously done in the flat space context in many papers, including [24–33]. This breaks all the supersymmetries. In flat space, the result was a tachyon in the spectrum. Understanding this closed string tachyon is a very interesting problem which has been addressed in e.g. [33–43]. A closed string tachyon is similarly expected here, the difference being that there should also be a dual CFT description. This could be a good way to get some control over closed string tachyon condensation.

The paper is organized as follows. In Sec. II the supersymmetric type IIB pp wave is reviewed with an emphasis on the symmetries, their algebra and the introduction of a convenient coordinate system for use in most of the paper. These results are used in Sec. III to identify spacelike Killing vectors and study the supersymmetries preserved by compactification along their orbits. The Green-Schwarz string is quantized on these geometries in Sec. IV. The SO(8) symmetry of the metric means that the quantization of the bosonic part of the string is independent of the choice of spacelike Killing vector. For the fermionic part of the string, we analyze the maximally supersymmetric compactification in Sec. IV B 1 and the minimally (half) supersymmetric S^1 compactification in Sec. IV B 2. In the latter case, the Hamiltonian is time dependent. The effect on the compactification of adding a twist is analyzed in Sec. V. Then T duality and supersymmetry is discussed in Sec. VI. Finally the $\mathrm{AdS}_5\times S^5$ parent orbifold, and the choices therein, is described in Sec. VII. Section VIII contains some conclusions. Two Appendices contain some additional useful formulas. Although most of the text is devoted to compactification on a single circle, in many cases, the results generalize to higher tori in an obvious way. One exception is a compactification on a T^2 involving the same two coordinates (in the "standard" coordinate system of [1]). Appendix A presents a (singular) coordinate system adapted to this compactification. In Appendix B some additional expressions for the Killing spinors are given.

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II. THE *pp* WAVE

In this section some useful facts about the pp-wave geometry, its isometries and its Killing spinors are collected. The supergravity solution is [1]

$$ds^2=2dx^+dx^- -4\mu^2 x^i x^i (dx^+)^2+dx^i dx^i, \quad (2.1a)$$

$$^{(5)}F=\mu dx^+(dx^1dx^2dx^3dx^4+dx^5dx^6dx^7dx^8), \quad (2.1b)$$

where $i=1\ldots 8$, the constant $\mu\neq 0$ will be kept arbitrary although it can be set to any convenient value by a coordinate transformation, and ${}^{(5)}F$ is the self dual Ramond-Ramond (RR) five-form field strength. All other fields vanish. These satisfy the type IIB equations of motion, including

$$G_{\mu\nu}=\frac{4}{3}F_{\mu\alpha\beta\gamma\delta}F_{\nu}^{\alpha\beta\gamma\delta}. \quad (2.2)$$

Greek indices run over all the coordinates.

It will be convenient to make the change of coordinates,

$$x^+=X^+,\quad x^-=X^- -2\mu X^1X^2,\quad x^I=X^I,\quad I=3\ldots 8 \quad (2.3a)$$

$$x^1=X^1\cos(2\mu X^+)-X^2\sin(2\mu X^+),$$
$$x^2=X^1\sin(2\mu X^+)+X^2\cos(2\mu X^+), \quad (2.3b)$$

Then the metric takes the form

$$ds^2=2dX^+dX^- -4\mu^2X^IX^I(dX^+)^2-8\mu X^2dX^1dX^+ + dX^idX^i, \quad (2.4a)$$

$$^{(5)}F=\mu dX^+(dX^1dX^2dX^3dX^4+dX^5dX^6dX^7dX^8). \quad (2.4b)$$

In this coordinate system, $\partial_1=\partial/\partial X^1$ is a manifest isometry. This is one of the circles on which we will consider compactification. In this coordinate system, X^1, X^2 are no longer massive bosons but instead have the quantum mechanics of a particle in a plane with constant magnetic field.

A. Isometries and Killing vectors

There is an obvious $SO(4)\times SO(4)\rtimes\mathbb{Z}_2$ symmetry which rotates and exchanges the $\{x^1,x^2,x^3,x^4\}$ and $\{x^5,x^6,x^7,x^8\}$ subspaces. In fact, 30 Killing vectors were identified in [1]: namely,

$$k_{e_+}=-\partial_+,\quad k_{e_-}=-\partial_-, \quad (2.5a)$$

$$k_{e_i}=-\cos(2\mu x^+)\partial_i-2\mu\sin(2\mu x^+)x^i\partial_-, \quad (2.5b)$$

$$k_{e_i^*}=-2\mu\sin(2\mu x^+)\partial_i+4\mu^2\cos(2\mu x^+)x^i\partial_-, \quad (2.5c)$$

$$k_{M_{ij}}=x^i\partial_j-x^j\partial_i,\text{both } i,j=1\ldots 4 \text{ or both } i,j=5\ldots 8. \quad (2.5d)$$

These obey the algebra

$$[k_{e_-},\text{all}]=0,\quad [k_{e_+},M_{ij}]=0, \quad (2.6a)$$

$$[k_{e_+},k_{e_i}]=k_{e_i^*},\quad [k_{e_+},k_{e_i^*}]=-4\mu^2k_{e_i},$$
$$[k_{e_i},k_{e_j^*}]=4\mu^2\delta_{ij}k_{e_-}, \quad (2.6b)$$

$$[k_{M_{ij}},k_{e_k}]=\delta_{jk}k_{e_i}-\delta_{ik}k_{e_j},$$
$$[k_{M_{ij}},k_{e_k^*}]=\delta_{jk}k_{e_i^*}-\delta_{ik}k_{e_j^*}, \quad (2.6c)$$

$$[k_{M_{ij}},k_{M_{kl}}]=\delta_{jk}k_{M_{il}}-\delta_{ik}k_{M_{jl}}-\delta_{jl}k_{M_{ik}}+\delta_{il}k_{M_{jk}}. \quad (2.6d)$$

In particular, M_{ij} are the $SO(4)\times SO(4)$ rotational generators.

B. Killing spinors and the superalgebra

Before listing the Killing spinors, some notation must be introduced. The ten dimensional Γ matrices obey

$$\{\Gamma^{\check\mu},\Gamma^{\check\nu}\}=2\eta^{\check\mu\check\nu},\quad \Gamma^{\check\pm}=\frac{1}{\sqrt{2}}(\Gamma^{\check 9}\pm\Gamma^{\check 0}), \quad (2.7)$$

where $\eta^{\check\mu\check\nu}$ is the mostly positive Minkowski metric of the tangent bundle. $\Gamma^{\mu_1\cdots\mu_p}$ are products of the Γ matrices, antisymmetrized with unit weight. The zenbein is [1]

$$e^{\check -}=dx^- -2\mu^2(x^i)^2dx^+,\quad e^{\check +}=dx^+,\quad e^{\check i}=dx^i. \quad (2.8)$$

I have used the cup to emphasize the tangent-space labels; this is important as the following formulas employ this choice of local frame. It is convenient to define

$$I=\Gamma^{\check 1}\Gamma^{\check 2}\Gamma^{\check 3}\Gamma^{\check 4},\quad J=\Gamma^{\check 5}\Gamma^{\check 6}\Gamma^{\check 7}\Gamma^{\check 8}. \quad (2.9)$$

In terms of the one-form $\Omega_\mu=(i/24)F_{\mu\alpha\beta\gamma\delta}\Gamma^{\alpha\beta\gamma\delta}$,

$$\Omega_-=0,\quad \Omega_+=\mu(I+J),$$
$$\Omega_i=\begin{cases}-\mu\Gamma^{\check +}\Gamma^{\check i}I, & i=1\ldots 4,\\ -\mu\Gamma^{\check +}\Gamma^{\check i}J, & i=5\ldots 8,\end{cases} \quad (2.10)$$

to each constant, complex positive chirality spinor ψ is associated the Killing spinor [1]

$$\epsilon(\psi)=[1-ix^i\Omega_i][\cos(\mu x^+)\mathbb{1}-i\sin(\mu x^+)I]\times[\cos(\mu x^+)\mathbb{1}-i\sin(\mu x^+)J]\psi. \quad (2.11)$$

These obey the Killing spinor equation

$$\left(\nabla_\mu+\frac{i}{24}F_{\mu\alpha\beta\gamma\delta}\Gamma^{\alpha\beta\gamma\delta}\right)\epsilon(\psi)=0. \quad (2.12)$$

Since there are 32 linearly independent ψs, there are 32 linearly independent positive chirality Killing spinors, corresponding to $\mathcal{N}=2B$ supersymmetry in ten dimensions.

These are acted on nontrivially by the Killing vectors. Specifically, the Lie derivative on spinors is defined by (see [44] and references therein)

$$\pounds_k \epsilon = k^\mu \nabla_\mu \epsilon + \frac{1}{4}(\nabla_\mu k_\nu)\Gamma^{\mu\nu}\epsilon. \tag{2.13}$$

Then [1,5]

$$\pounds_{k_{e_-}}\epsilon(\psi)=0, \quad \pounds_{k_{e_+}}\epsilon(\psi)=\epsilon(i\mu(I+J)\psi),$$

$$\pounds_{M_{ij}}\epsilon(\psi)=\epsilon\left(\frac{1}{2}\Gamma^{\check{i}\check{j}}\psi\right), \tag{2.14a}$$

$$\pounds_{k_{e_{i=1\dots4}}}\epsilon(\psi)=\epsilon(-i\mu I\Gamma^{\check{i}}\Gamma^{\check{+}}\psi)$$

$$\pounds_{k_{e^*_{i=1\dots4}}}\epsilon(\psi)=\epsilon(2\mu^2\Gamma^{\check{i}}\Gamma^{\check{+}}\psi) \tag{2.14b}$$

$$\pounds_{k_{e_{i=5\dots8}}}\epsilon(\psi)=\epsilon(-i\mu J\Gamma^{\check{i}}\Gamma^{\check{+}}\psi)$$

$$\pounds_{k_{e^*_{i=5\dots8}}}\epsilon(\psi)=\epsilon(2\mu^2\Gamma^{\check{i}}\Gamma^{\check{+}}\psi). \tag{2.14c}$$

This information will be used to count the number of supersymmetries preserved by compactification.

III. S^1 AND T^d COMPACTIFICATION OF THE *pp* WAVE

In [1], the authors considered compactification along the circle generated by the [manifest in Eqs. (2.1a), (2.1b)] isometry $k_{e_-}+k_{e_+}$.[1] However, that is not spacelike:

$$\|k_{e_-}+k_{e_+}\|^2=2-4\mu^2(x^i)^2. \tag{3.1}$$

Though compactification of timelike circles has been considered in [45], for example, such a circle is likely to introduce complications that I do not want to consider here.[2] Also, the isometry (3.1) breaks all the supersymmetries, except at special values of the radius. Instead, there are several isometries that, while not yet manifest, are spacelike of unit norm. Namely, define

$$k_{S^\pm_{ij}}=k_{e_i}\pm\frac{1}{2\mu}k_{e^*_j}, \tag{3.2}$$

and note that

$$\|k_{S^\pm_{ij}}\|^2=1, i\neq j. \tag{3.3}$$

[1]This was also one of the two isometries that was compactified in [10], in the context of the Penrose limit of $AdS_3\times S^3\times T^4$. The other isometry considered in [10] is one associated with the T^4.

[2]After much of this paper was written, [16] appeared which discusses this type of compactification with interesting results.

We will use this set of isometries to compactify the *pp*-wave geometry. [For fixed i, $\|k_{S^\pm_{ii}}\|^2=1\pm\sin 4\mu x^+$ goes null at $x^+=(2n\mp1)\pi/8\mu$.] Note that, in contrast to toroidal compactification of flat space, the isometries (3.2) are not hypersurface orthogonal.

Although there are many of these isometries, the $SO(4)\times SO(4)\rtimes\mathbb{Z}_2$ symmetry[3] implies that we can, without loss of generality, choose the "+" sign and $i=1$; then there are only two distinct choices of j, namely $j=2$ or $j=5$. Although the metric behavior is the same for both choices, the RR field is quite different.

For the first choice, $j=2$, the isometry $k_{S^+_{12}}$ is manifest in Eqs. (2.4a), (2.4b). Also,

$$\frac{\partial}{\partial X^+}=-k_{e_+}+2\mu k_{M_{12}} \tag{3.4}$$

is a Killing vector of the geometry. For $j=5$ the RR field is more complicated in manifest coordinates:

$$ds^2=2dX^+dX^--4\mu^2X^{\hat{I}}X^{\hat{I}}(dX^+)^2-8\mu X^5dX^1dX^+ + dX^idX^i, \tag{3.5a}$$

$$\begin{aligned} {}^{(5)}F=\mu dX^+[&\cos(2\mu X^+)dX^1dX^2dX^3dX^4\\ &-\sin(2\mu X^+)dX^5dX^2dX^3dX^4\\ &+\sin(2\mu X^+)dX^1dX^6dX^7dX^8\\ &+\cos(2\mu X^+)dX^5dX^6dX^7dX^8], \end{aligned} \tag{3.5b}$$

where $\hat{I}=2,3,4,6,7,8$. In particular note that

$$\frac{\partial}{\partial X^+}=\frac{\partial}{\partial x^+}+2\mu\left(x^1\frac{\partial}{\partial x^5}-x^5\frac{\partial}{\partial x^1}\right) \tag{3.6}$$

is not an isometry of the field configuration. For this choice of circle compactification, then, the light-cone Hamiltonian will be time dependent.

Focusing on the S^+_{12} compactification, the nine dimensional field configuration is easily read off [46] as

$$ds_9^2=2dX^+dX^--4\mu^2[4(X^2)^2+X^{\hat{I}}X^{\hat{I}}](dX^+)^2+dX^{\hat{i}}dX^{\hat{i}}, \tag{3.7a}$$

$$A_9=-4\mu X^2dX^+, \tag{3.7b}$$

$${}^{(3)}A_9=-\mu X^+dX^2dX^3dX^4, \tag{3.7c}$$

where $\hat{i}=2\dots8$, A_9 is the KK gauge field and ${}^{(3)}A_9$ is the dimensionally reduced potential for the ten dimensional self dual five-form field strength.

[3]Including the $\mathbb{Z}_2\in SO(4)$ element $x^i,x^j\to -x^i,-x^j$ (with $i,j=1\dots4$ or $i,j=5\dots8$) which takes $S^\pm_{ij}\to -S^\pm_{ij}$, $S^\pm_{ik}\to -S^\mp_{ik}$ and $S^\pm_{kj}\to S^\mp_{kj}$, $k\neq i,j$.

All of the $S^{\pm}_{ij}$ compactifications *preserve* at least 1/2 of the supersymmetry, or 16 supercharges. If i,j are in the same $SO(4)$ subgroup, then 3/4, or 24 supercharges, is preserved. This is seen from Eqs. (2.14a)–(2.14c), since

$$\pounds_{k_{e_i}+(1/2\mu)k_{e^*_j}}\epsilon(\psi)=\epsilon(i\mu Q^{ij}\psi), \tag{3.8}$$

where

$$Q^{ij}=\begin{cases}\Gamma^{\check{j}}\left(\frac{1}{2}\sum_{k,l=1}^{4}\epsilon_{ijkl}\Gamma^{\check{k}\check{l}}-i\mathbb{1}\right)\Gamma^{\check{+}}, & i,j=1\dots4,\\ \Gamma^{\check{j}}\left(\frac{1}{2}\sum_{k,l=5}^{8}\epsilon_{ijkl}\Gamma^{\check{k}\check{l}}-i\mathbb{1}\right)\Gamma^{\check{+}}, & i,j=5\dots8,\\ -\Gamma^{\check{j}}\left(\frac{1}{6}\sum_{k,l,m=1}^{4}\epsilon_{iklm}\Gamma^{\check{k}\check{l}\check{m}}\Gamma^{\check{j}}+i\mathbb{1}\right)\Gamma^{\check{+}}, & \\ \quad i=1\dots4, j=5\dots8, & \\ -\Gamma^{\check{j}}\left(\frac{1}{6}\sum_{k,l,m=5}^{8}\epsilon_{iklm}\Gamma^{\check{k}\check{l}\check{m}}\Gamma^{\check{j}}+i\mathbb{1}\right)\Gamma^{\check{+}}, & \\ \quad i=5\dots8, j=1\dots4. & \end{cases} \tag{3.9}$$

For the periodic spin structure, the supersymmetries preserved by the compactification are those whose Killing spinors are preserved by the isometry. That is, we want to count the number of complex, constant, positive chirality spinors annihilated by Q. [In principle, we should require only $e^{2\pi iR\pounds_k}\epsilon(\psi)=\epsilon(\psi)$; however, since Q^{ij} is nilpotent, this is the same thing. In particular, there are no special radii of enhanced supersymmetry.] Clearly, these include the 16 spinors annihilated by $\Gamma^{\check{+}}$. For i,j in different $SO(4)$ subgroups—the last two lines of Eq. (3.9)—the matrix in parenthesis is nondegenerate, so there are no other supersymmetries. When i,j are in the same $SO(4)$ grouping, then the matrix in parenthesis annihilates an additional 8 spinors. In this latter case,

$$P^{ij}=\mathbb{P}_+-\frac{1}{8}Q^{ij\dagger}Q^{ij}\mathbb{P}_+, \qquad [\text{no sum}; i,j \text{ in the same SO(4) grouping}] \tag{3.10}$$

is the projection operator onto these 24 spinors, where $\mathbb{P}_+$ is the projection onto positive chirality spinors.

More generally, we can consider other linear combinations of Killing vectors. However, from the previous analysis it is easy to see that generic linear combinations of k_{e_i} and $k_{e^*_j}$ will break all the supersymmetries. Although the addition of any multiple of k_{e_-} to, say, $k_{S^+_{12}}$ does not affect the norm—that is, $k_{S^+_{12}}+\alpha k_{e_-}$ provides a nice spacelike isometry—such an addition can be removed via a coordinate transformation generated by $k_{S^-_{21}}$; thus there is no need to consider this linear combination. Including k_{e_+} or $k_{M_{ij}}$ would generically break all the supersymmetries (though we will consider the latter in Sec. V). This leaves one interesting possibility:

$$\alpha k_{S^+_{12}}+\beta k_{S^+_{56}}, \quad \alpha^2+\beta^2=1, \tag{3.11}$$

has unit norm and preserves 20 supercharges, namely those preserved by the projection $P^{12}P^{56}$.

Finally, note that if we compactify simultaneously on the circles generated by both $k_{S^+_{ij}}$ and $k_{S^-_{ij}}$, then we break half the supersymmetry; namely the half not annihilated by $\Gamma^{\check{+}}$. This is also true for compactification with respect to both $k_{S^+_{ij}}$ and $k_{S^+_{ji}}$. For simultaneous compactification on, say, $k_{S^+_{ij}}$ and $k_{S^+_{ik}}$, $k\neq j$, again half the supersymmetry is broken. Although $k_{S^-_{ji}}$ preserves the same supersymmetries as $k_{S^+_{ij}}$, the two do not commute, and so the compactification on both does not correspond to a T^2 compactification. More specifically, in the coordinates (2.3a), (2.3b), if S^+_{ij} is compactified on a circle of radius R_1 and S^-_{ji} is compactified on a circle of radius R_2, then we must also identify the null coordinate $X^-\sim X^-+8\pi\mu R_1R_2$.

Thus, we can compactify on at most three circles before we have, at most, sixteen supercharges. The coordinate transformation which manifests the circles is an obvious generalization of Eqs. (2.3a), (2.3b), and in this coordinate system we have obvious generalizations of Eqs. (2.4a), (2.4b), and (3.5a), (3.5b).

The compactification also breaks some bosonic symmetry. For compactification on S^+_{12}, the $SO(4)\times SO(4)$ is broken to $SO(2)\times SO(4)$, and for the compactification on S^+_{15}, the residual rotational symmetry is $SO(3)\times SO(3)$. Also, though k_{e_+}, k_{e_j}, $k_{e^*_i}$ are no longer isometries after compactification on the orbit of S^+_{ij}, the linear combinations $k_{e_+}-2\mu k_{M_{12}}$ and $k_{e_j}+(1/2\mu)k_{e^*_i}$ are still isometries. Thus, the residual isometry group is 23 $[g\rtimes(SO(3)\times SO(3))]$ or 24-dimensional $[g\rtimes(SO(2)\times SO(4))]$ where g is the 17-dimensional group generated by the noncompact Killing vectors k_{e_-}, $k_{e_+}-2\mu k_{M_{12}}$, S^+_{ji}, $e_{k\neq j}$, $e^*_{k\neq i}$. For each additional compactified direction, the rotational group decreases further (to what depends on the details of the compactification; see Table I), but only removes one additional generator from g. This is summarized in Table I.

IV. THE GREEN-SCHWARZ STRING ON THE COMPACTIFIED *pp* WAVE

In this section the Green-Schwarz string is quantized on the compactified *pp* wave. The focus is on the compactification along the orbit of $k_{S^+_{12}}$; that is, $X^1\sim X^1+2\pi R$ for the solution (2.4a), (2.4b). Section IV B 2 will briefly discuss the S^+_{15} compactification. The bosons, of course, are impervious to the difference between S^+_{12} and S^+_{15}.

TABLE I. A summary of (many of) the independent ways in which one can compactify on commuting isometries, and the corresponding unbroken rotational symmetries and the number of unbroken supercharges. Not all compactifications preserving 16 supercharges are listed. The isometries are labeled as in Eq. (3.2). Only the continuous part of the rotational symmetries is listed.

Dimensions compactified	Isometries	Continuous rotations	Number of other bosonic generators	Preserved supercharges
	S_{12}^+	SO(2)×SO(4)		24
1	$\alpha S_{12}^+ + \beta S_{56}^+$	SO(2)×SO(2)	17	20
	S_{15}^+	SO(3)×SO(3)		16
	S_{12}^+, S_{34}^+	SO(4)		20
	S_{12}^+, S_{56}^+	SO(2)×SO(2)		20
	$\alpha S_{12}^+ + \beta S_{56}^+,\ S_{34}^+$	SO(2)		18
	S_{12}^+, S_{35}^+	SO(3)		16
	S_{15}^+, S_{26}^+	SO(2)×SO(2)		16
	S_{12}^+, S_{21}^+	SO(2)×SO(4)		16
2	S_{12}^+, S_{12}^-	SO(2)×SO(4)	16	16
	S_{12}^+, S_{13}^+	SO(4)		16
	S_{12}^+, S_{15}^+	SO(2)×SO(3)		16
	S_{15}^+, S_{51}^+	SO(3)×SO(3)		16
	S_{15}^+, S_{15}^-	SO(3)×SO(3)		16
	$S_{12}^+, S_{34}^+, S_{56}^+$	SO(2)		18
	$S_{12}^+, S_{35}^+, S_{46}^+$	SO(2)		16
	$S_{12}^+, S_{35}^+, S_{67}^+$	—		16
	$S_{15}^+, S_{26}^+, S_{37}^+$	—		16
	$S_{12}^+, S_{13}^+, S_{56}^+$	SO(2)		16
	$S_{12}^+, S_{13}^+, S_{45}^+$	SO(3)		16
	$S_{12}^+, S_{32}^+, S_{56}^+$	SO(2)		16
3	$S_{12}^+, S_{32}^+, S_{45}^+$	SO(3)	15	16
	$S_{12}^+, S_{32}^+, S_{34}^+$	SO(4)		16
	$S_{12}^+, S_{12}^-, S_{34}^+$	SO(4)		16
	$S_{12}^+, S_{12}^-, S_{56}^+$	SO(2)×SO(2)		16
	$S_{12}^+, S_{12}^-, S_{35}^+$	SO(3)		16
	$S_{12}^+, S_{35}^+, S_{35}^-$	SO(3)		16
	$S_{12}^+, S_{21}^+, S_{34}^+$	SO(4)		16
	$S_{12}^+, S_{21}^+, S_{56}^+$	SO(2)×SO(2)		16
	$S_{12}^+, S_{21}^+, S_{35}^+$	SO(3)		16
	$S_{12}^+, S_{35}^+, S_{53}^+$	SO(3)		16
$d \geq 4$	many choices	various	$18-d$	16

A. Bosonic oscillations

The world sheet coordinates are τ,σ and the Lorentzian world sheet metric is γ_{ab}. Light-cone gauge is given[4] by setting $X^+=\tau$, $\det\gamma=-1$, $\partial_\sigma\gamma_{\sigma\sigma}=0$ and $\gamma_{\tau\sigma}=0$. The bosonic action in light-cone gauge is then

$$S_{\mathrm{B}}=-\frac{1}{4\pi\alpha'}\int d^2\sigma\{-\gamma_{\sigma\sigma}[2\dot{X}^- -4\mu^2(X^I)^2-8\mu X^2\dot{X}^1 +(\dot{X}^i)^2]+\gamma_{\sigma\sigma}^{-1}(X^{i\prime})^2\}, \tag{4.1}$$

where overdots and primes denote differentiation with respect to τ and σ, respectively. This is supplemented with the constraint equation, of which the bosonic part [see also Eq. (4.17)] is

$$X^{-\prime}=4\mu X^2 X^{1\prime}-\dot{X}^i X^{i\prime}. \tag{4.2}$$

We see that, as usual, $p_-=\delta S/\delta\dot{X}^-=\gamma_{\sigma\sigma}/2\pi\alpha'$, along with the equation [which also gets a fermionic contribution in Eq. (4.18)]

$$\dot{X}^-=4\mu X^2\dot{X}^1+2\mu^2X^IX^I-\frac{1}{2}(\dot{X}^i)^2-\frac{1}{2p_-^2\alpha'^2}(X^{i\prime})^2. \tag{4.3}$$

The equation of motion for X^I is the same massive equation as for the uncompactified pp wave [5,6]. For $X=X^1+iX^2$,

$$\ddot{X}-c^2X''=-4\mu i\dot{X}, \tag{4.4a}$$

$$\ddot{\bar{X}}-c^2\bar{X}''=4\mu i\dot{\bar{X}}, \tag{4.4b}$$

[4] Light-cone gauge is allowed since X^+ obeys a harmonic equation of motion before gauge fixing.

and $X \sim X + 2\pi R$, where $c = (\alpha' p_-)^{-1}$. Thus the mode expansion is

$$X = X_0 e^{-2\mu i\tau}\cos 2\mu\tau + \frac{p_0}{2\mu p_-} e^{-2\mu i\tau}\sin 2\mu\tau + wR\sigma + \frac{i}{p_-\sqrt{\alpha'}}\sum_{n\neq 0}\frac{e^{-2\mu i\tau}}{n\sqrt{c^2+4\frac{\mu^2}{n^2}}}\left[\alpha_n e^{-in(\sqrt{c^2+4\mu^2/n^2}\tau+\sigma)} + \tilde{\alpha}_n e^{-in(\sqrt{c^2+4\mu^2/n^2}\tau-\sigma)}\right], \tag{4.5a}$$

$$\bar{X} = \bar{X}_0 e^{2\mu i\tau}\cos 2\mu\tau + \frac{\bar{p}_0}{2\mu p_-} e^{2\mu i\tau}\sin 2\mu\tau + wR\sigma + \frac{i}{p_-\sqrt{\alpha'}}\sum_{n\neq 0}\frac{e^{2\mu i\tau}}{n\sqrt{c^2+4\frac{\mu^2}{n^2}}}\bar{\alpha}_n e^{-in(\sqrt{c^2+4\mu^2/n^2}\tau+\sigma)} + \bar{\tilde{\alpha}}_n e^{-in(\sqrt{c^2+4\mu^2/n^2}\tau-\sigma)}, \tag{4.5b}$$

$$X^I = X_0^I\cos 2\mu\tau + \frac{p_0^I}{2\mu p_-}\sin 2\mu\tau + \frac{i}{p_-\sqrt{\alpha'}}\sum_{n\neq 0}\frac{1}{n\sqrt{c^2+4\frac{\mu^2}{n^2}}}\left[\alpha_n^I e^{-in(\sqrt{c^2+4\mu^2/n^2}\tau+\sigma)} + \tilde{\alpha}_n^I e^{-in(\sqrt{c^2+4\mu^2/n^2}\tau-\sigma)}\right]. \tag{4.5c}$$

Note that $\alpha_n, \tilde{\alpha}_n, \ldots$ is a positive (negative) frequency mode for n positive (negative) (and large). The mode expansion for $X,\bar{X}$ is reminiscent of the mode expansion for the massive scalar, but is rotated by an extra time dependent phase factor $e^{2\mu i\tau}$. The canonical commutation relations are

$$[X_0,\bar{p}_0] = i = [\bar{X}_0,p_0] \quad [X_0^I,p_0^J] = i\delta^{IJ}, \tag{4.6a}$$

$$[\alpha_n,\bar{\alpha}_m] = 2n\sqrt{1+4\frac{\mu^2}{n^2c^2}}\delta_{n,-m} = [\tilde{\alpha}_n,\bar{\tilde{\alpha}}_m], \tag{4.6b}$$

$$[\alpha_n^I,\alpha_m^J] = n\sqrt{1+4\frac{\mu^2}{n^2c^2}}\delta^{IJ}\delta_{n,-m} = [\tilde{\alpha}_n^I,\tilde{\alpha}_m^J], \tag{4.6c}$$

and all others vanish. Finally, note that

$$X_0^\dagger = \bar{X}_0, \quad p_0^\dagger = \bar{p}_0, \quad p_0^{I\dagger} = p_0^I, \quad X_0^{I\dagger} = X_0^I, \tag{4.7a}$$

$$\alpha_n^\dagger = \bar{\alpha}_{-n}, \quad \tilde{\alpha}_n^\dagger = \bar{\tilde{\alpha}}_{-n}, \quad \alpha_n^{I\dagger} = \alpha_{-n}^I, \quad \tilde{\alpha}_n^{I\dagger} = \tilde{\alpha}_{-n}^I. \tag{4.7b}$$

In terms of oscillators, the bosonic part of the Hamiltonian is (ignoring the zero-point energy, which supersymmetry cancels against the fermionic zero-point energy)

$$\begin{aligned} H_{\mathrm{B}} = {}& 2\mu^2 p_- X_0\bar{X}_0 + 2\mu^2 p_-(X_0^I)^2 + \frac{p_0\bar{p}_0}{2p_-} + \frac{(p_0^I)^2}{2p_-} \\ &+ \frac{(wR)^2}{2\alpha'^2 p_-} + c\sum_{n>0}(\alpha_{-n}\bar{\alpha}_n + \bar{\alpha}_{-n}\alpha_n + \tilde{\alpha}_{-n}\bar{\tilde{\alpha}}_n \\ &+ \bar{\tilde{\alpha}}_{-n}\tilde{\alpha}_n + 2\alpha_{-n}^I\alpha_n^I + 2\tilde{\alpha}_{-n}^I\tilde{\alpha}_n^I) - i\mu(X_0\bar{p}_0 - \bar{X}_0 p_0) \\ &+ c\sum_{n>0}\frac{2\mu}{n\sqrt{c^2+4\mu^2/n^2}}(\alpha_{-n}\bar{\alpha}_n - \bar{\alpha}_{-n}\alpha_n + \tilde{\alpha}_{-n}\bar{\tilde{\alpha}}_n \\ &- \bar{\tilde{\alpha}}_{-n}\tilde{\alpha}_n). \end{aligned} \tag{4.8}$$

Note that this is not the same Hamiltonian as in [5,6]; in this coordinate system that is natural for compactification, the Hamiltonian includes the angular momentum generator in the 12-plane—see Eq. (3.4).

B. Fermionic oscillations

The type IIB Green-Schwarz string is written in terms of a pair of positive chirality Majorana-Weyl space-time spinors Θ^Λ, $\Lambda = 1,2$. Along with the index Λ come the matrices

$$\rho_0 = i\sigma^2, \quad \rho_1 = \sigma^1, \quad \rho_3 = \sigma^3, \tag{4.9}$$

where the σ^i are the Pauli matrices. The starting point is the observation that with sufficient symmetry and the light-cone gauge fixing

$$\Gamma^{\check{+}}\Theta^\Lambda = 0, \tag{4.10}$$

one can immediately write the covariantized action [6,47,48]

$$S_{\mathrm{F}} = -\frac{1}{2\pi}\int d^2\sigma\sqrt{-\gamma}\, i(\gamma^{ab}\delta_{\Lambda\Sigma} + 2\pi\epsilon^{ab}\rho_{3\Lambda\Sigma})\partial_a X^\mu\bar{\Theta}^\Lambda\Gamma_\mu(\mathcal{D}_b\Theta)^\Sigma, \tag{4.11}$$

where, in the Majorana representation $\bar{\Theta}^\Lambda = (\Theta^\Lambda)^{\mathsf{T}}\Gamma^{\check{0}}$, and the covariant derivative on spinors is

$$\mathcal{D}_a = \partial_a + \frac{1}{4}\partial_a X^\mu\omega_{\mu\check{\sigma}\check{\tau}}\Gamma^{\check{\sigma}\check{\tau}} - \frac{1}{2\times 5!}{}^{(5)}F_{\mu\nu\lambda\sigma\tau}\Gamma^{\mu\nu\lambda\sigma\tau}\partial_a X^\rho\Gamma_\rho\rho_0, \tag{4.12}$$

where $\omega_{\mu\check{\sigma}\check{\tau}}$ is the spacetime spin connection and I have included the effect of a background five-form field strength, but no other background fields. (See e.g. [6] for a more complete expression.) The world sheet metric is that of Sec. IV A and ϵ^{ab} is a true tensor with $\epsilon_{\tau\sigma} = \sqrt{-\gamma} = 1$.

1. Compactification on S^+_{12}

The natural coordinate system for compactification along $k_{S^+_{12}}$ is that of Eq. (2.4), for which the zenbein is

$$e^{\check{-}}=dX^- -2\mu^2(X^I)^2dX^+ -4\mu X^2 dX^1,$$

$$e^{\check{+}}=dX^+,\quad e^{\check{i}}=dX^i, \tag{4.13}$$

and the nonzero components of the spin connection are

$$\omega_{+\check{+}\check{I}}=-4\mu^2X^I,\quad \omega_{1\check{+}\check{2}}=-2\mu=-\omega_{2\check{+}\check{1}}=\omega_{+\check{1}\check{2}}. \tag{4.14}$$

In particular, note that, for this geometry, four-and-higher-Fermi Riemann curvature terms do not appear in Eq. (4.11), due to the gauge fixing (4.10).

Thus for the background (2.4a), (2.4b), the fermionic action is

$$S_{\mathrm{F}}=i\alpha' p_-\int d^2\sigma[\bar{\Theta}^\Lambda\Gamma^{\check{-}}(\delta_{\Lambda\Sigma}\partial_\tau+c\rho_{3\Lambda\Sigma}\partial_\sigma)\Theta^\Sigma -2\mu\bar{\Theta}^\Lambda\Gamma^{\check{-}}I\rho_{0\Lambda\Sigma}\Theta^\Sigma-\mu\bar{\Theta}^\Lambda\Gamma^{\check{-}\check{1}\check{2}}\Theta^\Lambda]. \tag{4.15}$$

This leads to the equations of motion

$$(\partial_\tau+c\partial_\sigma-\mu\Gamma^{\check{1}\check{2}})\Theta^1-2\mu I\Theta^2=0, \tag{4.16a}$$

$$(\partial_\tau-c\partial_\sigma-\mu\Gamma^{\check{1}\check{2}})\Theta^2+2\mu I\Theta^1=0. \tag{4.16b}$$

Also the constraint equation (4.2) should be replaced with

$$X^{-\prime}=4\mu X^2X^{1\prime}-\dot{X}^iX^{i\prime}-i\alpha'\bar{\Theta}^\Lambda\Gamma^{\check{-}}\partial_\sigma\Theta^\Lambda, \tag{4.17}$$

and Eq. (4.3) gets a fermionic contribution so that it reads

$$\dot{X}^-=4\mu X^2\dot{X}^1+2\mu^2X^IX^I-\frac{1}{2}(\dot{X}^i)^2-\frac{1}{2p_-^2\alpha'^2}(X^{i\prime})^2 -i\alpha'\bar{\Theta}^\Lambda\Gamma^{\check{-}}\partial_\tau\Theta^\Lambda+i\alpha'\bar{\Theta}^\Lambda\Gamma^{\check{-}}\Gamma^{\check{1}\check{2}}\Theta^\Lambda +2i\alpha'\bar{\Theta}^\Lambda\Gamma^{\check{-}}I\rho_{0\Lambda\Sigma}\Theta^\Sigma. \tag{4.18}$$

The general solution to the equations of motion (4.16), subject to the periodicity condition

$$\Theta^\Lambda(\sigma+2\pi,\tau)=\Theta^\Lambda(\sigma,\tau), \tag{4.19}$$

is

$$\Theta^1=\sqrt{\frac{c}{2\pi}}e^{\mu\Gamma^{\check{1}\check{2}}\tau}(\cos 2\mu\tau)\theta_0^1 +\sqrt{\frac{c}{2\pi}}e^{\mu\Gamma^{\check{1}\check{2}}\tau}(\sin 2\mu\tau)I\theta_0^2 +\sqrt{\frac{c}{2\pi}}\sum_{n\neq 0}\frac{e^{\mu\Gamma^{\check{1}\check{2}}\tau}}{\sqrt{1+\left(\frac{n(\sqrt{c^2+4\mu^2/n^2}-c)}{2\mu}\right)^2}} \times\left[e^{-in(\sqrt{c^2+4\mu^2/n^2}\tau-\sigma)}\theta_n^1+\frac{i}{2\mu}n\left(\sqrt{c^2+\frac{4\mu^2}{n^2}}-c\right) \times e^{-in(\sqrt{c^2+4\mu^2/n^2}\tau+\sigma)}I\theta_n^2\right], \tag{4.20a}$$

$$\Theta^2=\sqrt{\frac{c}{2\pi}}e^{\mu\Gamma^{\check{1}\check{2}}\tau}(\cos 2\mu\tau)\theta_0^2 -\sqrt{\frac{c}{2\pi}}e^{\mu\Gamma^{\check{1}\check{2}}\tau}(\sin 2\mu\tau)I\theta_0^1 +\sqrt{\frac{c}{2\pi}}\sum_{n\neq 0}\frac{e^{\mu\Gamma^{\check{1}\check{2}}\tau}}{\sqrt{1+\left(\frac{n(\sqrt{c^2+4\mu^2/n^2}-c)}{2\mu}\right)^2}} \times\left[e^{-in(\sqrt{c^2+4\mu^2/n^2}\tau+\sigma)}\theta_n^2-\frac{i}{2\mu}n\left(\sqrt{c^2+\frac{4\mu^2}{n^2}}-c\right) \times e^{-in(\sqrt{c^2+4\mu^2/n^2}\tau-\sigma)}I\theta_n^1\right], \tag{4.20b}$$

where θ_n^Λ are positive chirality Majorana-Weyl spinor operators for which $\Gamma^{\check{+}}\theta_n^\Lambda=0$. Comparing to [5,6], we see that the fermions are rotated like the bosons, but in the spin representation, of course. Half of the θ_n^Λs (or four for each Λ) have $\Gamma^{\check{1}\check{2}}=i$ and the other half have $\Gamma^{\check{1}\check{2}}=-i$. Note that the periodicity condition (4.19) is unaffected by compactification in the X^1 direction, again due in part to the gauge fixing (4.10); thus Eqs. (4.20a), (4.20b) hold in winding sectors, as well.

Reality of Θ^Λ implies

$$\theta_n^{\alpha\Lambda\dagger}=\theta_{-n}^{\alpha\Lambda}, \tag{4.21}$$

as usual, where here, α is a spinor index; in particular, the zero modes are self-conjugate. The canonical commutation relations are

$$\{\theta_n^{\alpha\Lambda},\theta_m^{\beta\Sigma}\}=\frac{1}{\sqrt{2}}\delta_{n,-m}\frac{(\Gamma^{\check{+}}\Gamma^{\check{-}})^{\alpha\beta}}{2}\delta^{\Lambda\Sigma}, \tag{4.22}$$

where the Γ matrices enforce light-cone gauge by projecting onto the subspace annihilated by $\Gamma^{\check{+}}$.

The fermionic contribution to the Hamiltonian is (again ignoring the zero-point energy)

$$H_{\mathrm{F}}=i\mu\overline{\theta_0^1}\Gamma^{\check{-}\check{1}\check{2}}\theta_0^1+i\mu\overline{\theta_0^2}\Gamma^{\check{-}\check{1}\check{2}}\theta_0^2+4\mu i\overline{\theta_0^1}\Gamma^{\check{-}}I\theta_0^2 2\sum_{n>0}\left[\overline{\theta_{-n}^1}\Gamma^{\check{-}}\left(n\sqrt{c^2+\frac{4\mu^2}{n^2}}+i\mu\Gamma^{\check{1}\check{2}}\right)\theta_n^1+\overline{\theta_{-n}^2}\Gamma^{\check{-}}\left(n\sqrt{c^2+\frac{4\mu^2}{n^2}}+i\mu\Gamma^{\check{1}\check{2}}\right)\theta_n^2\right]. \tag{4.23}$$

Again, this is recognizable as the Hamiltonian of [5,6] plus fermionic angular momentum in the 12-plane.

2. Compactification on S^+_{15}

For compactification along the circle generated by $k_{S^+_{15}}$ the zenbein is

$$e^{\check{-}}=dX^- -2\mu^2(X^I)^2dX^+ -4\mu X^5dX^1,$$
$$e^{\check{+}}=dX^+,\quad e^{\check{i}}=dX^i, \tag{4.24}$$

and more importantly, the field strength (3.5b) has gained some X^+-dependence. As a result, the action (4.11) is now

$$S_{\mathrm{F}}=i\alpha' p_-\int d^2\sigma[\overline{\Theta}^\Lambda\Gamma^{\check{-}}(\delta_{\Lambda\Sigma}\partial_\tau+c\rho_{3\Lambda\Sigma}\partial_\sigma)\Theta^\Sigma -2\mu\overline{\Theta}^\Lambda\Gamma^{\check{-}}Ie^{-2\mu\tau\Gamma^{\check{1}\check{5}}}\rho_{0\Lambda\Sigma}\Theta^\Sigma-\mu\overline{\Theta}^\Lambda\Gamma^{\check{-}\check{1}\check{5}}\Theta^\Lambda]. \tag{4.25}$$

The action is now time dependent. Despite this difference, the classical solution to the equations of motion is still given by the mode expansion (4.20a), (4.20b) after replacing $\Gamma^{\check{1}\check{2}}$ with $\Gamma^{\check{1}\check{5}}$. Note that unlike there, the ordering of the Γ matrices is important here since $\Gamma^{\check{1}\check{5}}$ *anti*commutes with I. Explicitly,

$$\Theta^1=\sqrt{\frac{c}{2\pi}}e^{\mu\Gamma^{\check{1}\check{5}}\tau}(\cos 2\mu\tau)\theta_0^1-e^{\mu\Gamma^{\check{1}\check{5}}\tau}\sqrt{\frac{c}{2\pi}}(\sin 2\mu\tau)I\theta_0^2+\sqrt{\frac{c}{2\pi}}\sum_{n\neq 0}\frac{e^{\mu\Gamma^{\check{1}\check{5}}\tau}}{\sqrt{1+\left(\frac{n(\sqrt{c^2+4\mu^2/n^2}-c)}{2\mu}\right)^2}}\times\left[e^{-in(\sqrt{c^2+4\mu^2/n^2}\tau-\sigma)}\theta_n^1+\frac{i}{2\mu}n\left(\sqrt{c^2+\frac{4\mu^2}{n^2}}-c\right)\times e^{-in(\sqrt{c^2+4\mu^2/n^2}\tau+\sigma)}I\theta_n^2\right], \tag{4.26a}$$

$$\Theta^2=\sqrt{\frac{c}{2\pi}}e^{\mu\Gamma^{\check{1}\check{5}}\tau}(\cos 2\mu\tau)\theta_0^2+\sqrt{\frac{c}{2\pi}}e^{\mu\Gamma^{\check{1}\check{5}}\tau}(\sin 2\mu\tau)I\theta_0^1+\sqrt{\frac{c}{2\pi}}\sum_{n\neq 0}\frac{e^{\mu\Gamma^{\check{1}\check{5}}\tau}}{\sqrt{1+\left(\frac{n(\sqrt{c^2+4\mu^2/n^2}-c)}{2\mu}\right)^2}}\times\left[e^{-in(\sqrt{c^2+4\mu^2/n^2}\tau+\sigma)}\theta_n^2-\frac{i}{2\mu}n\left(\sqrt{c^2+\frac{4\mu^2}{n^2}}-c\right)\times e^{-in(\sqrt{c^2+4\mu^2/n^2}\tau-\sigma)}I\theta_n^1\right]. \tag{4.26b}$$

So, not surprisingly, we again obtain the rotated uncompactified result.

V. TWISTED COMPACTIFICATION

In this section the compactification is done along the isometry

$$-k_{S^+_{12}}+\frac{q}{R}k_{M_{34}}. \tag{5.1}$$

In flat space, this compactification leads to the Melvin universe—see e.g. [24–31].[5] I have chosen to rotate in the 34-plane as the circle is traversed, but little will change in the following [though the rotational symmetry of the resulting space would be $SO(2)\times SO(2)$ instead of $SO(4)$] if the rotation is, say, in the 56-plane instead. Note that—unless q is an even integer, at which point the theory is equivalent to compactification along $k_{S^+_{12}}$—all the supersymmetry is broken by this compactification.

With

$$Z=X^3+iX^4, \tag{5.2}$$

identification on the orbits of Eq. (5.1) is equivalent to the identification

$$\{X^1,Z,\overline{Z},\Theta^\Lambda\}\sim\{X^1+2\pi R,e^{2\pi iq}Z,e^{-2\pi iq}\overline{Z},e^{\pi q\Gamma^{\check{3}\check{4}}}\Theta^\Lambda\}. \tag{5.3}$$

The action is the same as Eqs. (4.1) and (4.15), but following the identification (5.3), the boundary conditions are replaced by

$$Z(\sigma+2\pi,\tau)=e^{2\pi iqw}Z(\sigma,\tau),$$
$$\Theta^\Lambda(\sigma+2\pi,\tau)=e^{\pi qw\Gamma^{\check{3}\check{4}}}\Theta^\Lambda(\sigma,\tau), \tag{5.4}$$

where w is the winding number in the mode expansion (4.5a)–(4.5c). The effect of the twist is therefore to shift the moding of Z and Θ, when $w\neq 0$. (If q is rational, the moding of Z is not shifted whenever $qw\in\mathbb{Z}$ and Θ modes are not shifted when $qw\in 2\mathbb{Z}$.) With the notation

[5] Compared to much of the literature, $q_{\mathrm{here}}=(qR)_{\mathrm{there}}$. In particular, q_{here} is dimensionless.

$$\hat{x} \equiv x - \lfloor x \rfloor = \text{the fractional part of } x, \tag{5.5}$$

then for $\widehat{qw} \neq 0$, the mode expansion (4.5c) is replaced by

$$Z = \frac{i}{p_- \sqrt{\alpha'}} \sum_{n\neq 0} \left[\frac{1}{(n-\widehat{qw})\sqrt{c^2 + 4\frac{\mu^2}{(n-\widehat{qw})^2}}} \beta_{n-\widehat{qw}} \times e^{-i(n-\widehat{qw})(\sqrt{c^2+[4\mu^2/(n-\widehat{qw})^2]}\tau+\sigma)} + \frac{1}{(n+\widehat{qw})\sqrt{c^2 + 4\frac{\mu^2}{(n+\widehat{qw})^2}}} \tilde{\beta}_{n+\widehat{qw}} \times e^{-i(n+\widehat{qw})(\sqrt{c^2+[4\mu^2/(n+\widehat{qw})^2]}\tau-\sigma)} \right], \tag{5.6a}$$

$$\bar{Z} = \frac{i}{p_- \sqrt{\alpha'}} \sum_{n\neq 0} \left[\frac{1}{(n+\widehat{qw})\sqrt{c^2 + 4\frac{\mu^2}{(n+\widehat{qw})^2}}} \bar{\beta}_{n+\widehat{qw}} \times e^{-i(n+\widehat{qw})(\sqrt{c^2+[4\mu^2/(n+\widehat{qw})^2]}\tau+\sigma)} + \frac{1}{(n-\widehat{qw})\sqrt{c^2 + 4\frac{\mu^2}{(n-\widehat{qw})^2}}} \bar{\tilde{\beta}}_{n-\widehat{qw}} \times e^{-i(n-\widehat{qw})(\sqrt{c^2+[4\mu^2/(n-\widehat{qw})^2]}\tau-\sigma)} \right], \tag{5.6b}$$

and if $\widehat{qw}/2 \neq 0$ the expansion (4.20a), (4.20b) is replaced by

$$\Theta^1(\sigma,\tau) = \sqrt{\frac{c}{2\pi}} \sum_{n\neq 0} e^{\mu \Gamma^{\breve{1}\breve{2}}\tau} \times \left[\frac{1}{\sqrt{1 + \left(\frac{\sqrt{(n - i(\widehat{qw}/2)\Gamma^{\breve{3}\breve{4}})^2 c^2 + 4\mu^2} - c}{2\mu} \right)^2}} e^{-i(n-i(\widehat{qw}/2)\Gamma^{\breve{3}\breve{4}})[\sqrt{c^2+[4\mu^2/(n-i(\widehat{qw}/2)\Gamma^{\breve{3}\breve{4}})^2]}\tau-\sigma]} \times \theta^1_{n-i(\widehat{qw}/2)\Gamma^{\breve{3}\breve{4}}} + \frac{i}{2\mu} \frac{(n+i(\widehat{qw}/2)\Gamma^{\breve{3}\breve{4}}) \left(\sqrt{c^2 + \frac{4\mu^2}{(n+i(\widehat{qw}/2)\Gamma^{\breve{3}\breve{4}})^2}} - c \right)}{\sqrt{1 + \left(\frac{\sqrt{(n + i(\widehat{qw}/2)\Gamma^{\breve{3}\breve{4}})^2 c^2 + 4\mu^2} - c}{2\mu} \right)^2}} \times e^{-i(n+i(\widehat{qw}/2)\Gamma^{\breve{3}\breve{4}})[\sqrt{c^2+[4\mu^2/(n+i(\widehat{qw}/2)\Gamma^{\breve{3}\breve{4}})^2]}\tau+\sigma]} I \theta^2_{n+i(\widehat{qw}/2)\Gamma^{\breve{3}\breve{4}}} \right], \tag{5.6c}$$

$$\Theta^2(\sigma,\tau)=\sqrt{\frac{c}{2\pi}}\sum_{n\neq 0} e^{\mu\Gamma^{\check{1}\check{2}}\tau}$$
$$\times\left[\frac{1}{\sqrt{1+\left(\frac{\sqrt{(n+i(\widehat{qw}/2)\Gamma^{\check{3}\check{4}})^2c^2+4\mu^2}-c}{2\mu}\right)^2}}e^{-i(n+i(\widehat{qw}/2)\Gamma^{\check{3}\check{4}})[\sqrt{c^2+[4\mu^2/(n+i(\widehat{qw}/2)\Gamma^{\check{3}\check{4}})^2]}\tau+\sigma]}\right.$$
$$\times\theta^2_{n+i(\widehat{qw}/2)\Gamma^{\check{3}\check{4}}}-\frac{i}{2\mu}\frac{(n-i(\widehat{qw}/2)\Gamma^{\check{3}\check{4}})\left(\sqrt{c^2+\frac{4\mu^2}{(n-i(\widehat{qw}/2)\Gamma^{\check{3}\check{4}})^2}}-c\right)}{\sqrt{1+\left(\frac{\sqrt{(n-i(\widehat{qw}/2)\Gamma^{\check{3}\check{4}})^2c^2+4\mu^2}-c}{2\mu}\right)^2}}$$
$$\left.\times e^{-i(n-i(\widehat{qw}/2)\Gamma^{\check{3}\check{4}})[\sqrt{c^2+[4\mu^2/(n-i(\widehat{qw}/2)\Gamma^{\check{3}\check{4}})^2]}\tau-\sigma]}I\theta^1_{n-i(\widehat{qw}/2)\Gamma^{\check{3}\check{4}}}\right]. \tag{5.6d}$$

Although the labels would make more sense in a (complex) basis of eigenspinors of $\Gamma^{\check{3}\check{4}}$, the expression is unambiguous. The Hermiticity properties and commutation relations are essentially the same as in Sec. IV B 1.

VI. *T* DUALITY

Performing a T duality of the nine dimensional geometry 3.7 along the X^1 direction leads to the type IIA configuration[6]

$$ds^2_{\mathrm{IIA}}=2dX^+dX^--4\mu^2[4(X^2)^2+(X^I)^2](dX^+)^2+(dX^i)^2, \tag{6.1a}$$

$$B=-4\mu X^2dX^1dX^+, \tag{6.1b}$$

$$^{(3)}A=8\mu X^+dX^2dX^3dX^4, \tag{6.1c}$$

where B is the Neveu-Schwarz (NS-NS) two-form and $^{(3)}A$ is the RR three-form potential. It is straightforward to check that these obey the type IIA equations of motion.

This can be further lifted to M theory, giving the field configuration

$$ds^2_{\mathrm{M}}=2dX^+dX^--4\mu^2[4(X^2)^2+(X^I)^2](dX^+)^2+(dX^i)^2+(dX^{11})^2, \tag{6.2a}$$

$$^{(3)}C=4\mu X^+dX^1dX^2dX^{11}+8\mu X^+dX^2dX^3dX^4. \tag{6.2b}$$

This solution preserves 24 supercharges. The M-theory Killing spinor equation is [44]

$$0=\mathcal{D}_\mu\epsilon\equiv\nabla_\mu\epsilon-\frac{1}{288}{}^{(4)}F_{\sigma\tau\lambda\rho}[\Gamma^{\sigma\tau\lambda\rho}\Gamma_\mu+4\Gamma^{\sigma\tau\lambda}\delta^\rho_\mu]. \tag{6.3}$$

The integrability condition, $[\mathcal{D}_\mu,\mathcal{D}_\nu]\epsilon=0$ gives $\Gamma^{\check{+}}(\Gamma^{\check{1}\check{3}\check{4}\check{11}}-\mathbb{1})\epsilon=0$. Since Eq. (6.3) is a first-order differential equation, this means that there are precisely 24 Killing spinors, namely

$$\epsilon(\psi)=\left[1+\sum_{i=2}^{8}X^i\tilde{\Omega}_i\right]\exp\left[-\frac{\mu}{3}X^+(3-\Gamma^{\check{-}}\Gamma^{\check{+}})\Gamma^2(2\Gamma^{\check{3}\check{4}}+\Gamma^{\check{1}\check{11}})\right]\psi, \tag{6.4}$$

with ψ a constant spinor obeying $(\Gamma^{\check{1}\check{3}\check{4}\check{11}}-\mathbb{1})\Gamma^{\check{+}}\psi=0$, and

$$\tilde{\Omega}_2=\frac{2}{3}\mu\Gamma^{\check{+}}(2\Gamma^{\check{3}\check{4}}-\Gamma^{\check{1}\check{11}}), \tag{6.5a}$$

$$\tilde{\Omega}_{I=3,4}=-\frac{1}{3}\mu\Gamma^{\check{+}\check{2}\check{I}}(4\Gamma^{\check{3}\check{4}}+\Gamma^{\check{1}\check{11}})$$

[6] The normalizations are that of [49], except that, $^{(5)}F_{\mathrm{here}}=\frac{1}{8}{}^{(5)}F_{[49]}$.

$$\tilde{\Omega}_{I=5,\ldots,8}=\frac{1}{3}\mu\Gamma^{\check{+}\check{2}\check{I}}(2\Gamma^{\check{3}\check{4}}-\Gamma^{\check{1}\check{1}\check{1}}). \tag{6.5b}$$

The continued presence of the 24 Killing spinors after T duality may not seem surprising, but it occurs only because the type IIB Killing spinors do not carry momentum—or equivalently, are independent of X^1, as is shown in Appendix B. Otherwise, some of the spinors would have winding and would not be visible in the T-dualized supergravity, as in [22] (see also the recent [16]). Observe also that these M-theory Killing spinors are similarly independent of both X^1 and X^{11}.

VII. THE $AdS_5 \times S^5$ ORIGIN OF THE CIRCLE

The pp wave arises from the Penrose limit of $AdS_5 \times S^5$ [2–4]. This is a powerful observation, as has been made clear by the CFT description of [4]. In [3] it was explained that the Penrose limit preserves the isometries of the original spacetime (though not the algebra). Since we have not gained any isometries by taking the Penrose limit, the isometry on which we are compactifying must correspond to some isometry of $AdS_5 \times S^5$; that is, our compactified spacetime is the Penrose limit of a quotient of $AdS_5 \times S^5$. That quotient will be identified here.

$AdS_5 \times S^5$ has isometry group $SO(4,2)\times SO(6)$, with generators $M_{\mu\nu}$ and P_μ, where $M_{\mu\nu}$ are rotations and P_μ are "translations" which commute to rotations. (For unity of the presentation, either both $\mu,\nu=0\ldots4$ or both $\mu,\nu=5\ldots9$.) In terms of embedding coordinates Y^M, where $\eta_{MN}Y^MY^N=\pm1$, the isometries are rotational: with $M_{MN}=Y^M\partial_N-Y^N\partial_M$ for $M,N=-1\ldots4$ or $M,N=5\ldots10$, P_μ is then $M_{-1,\mu}$ or $M_{10,\mu}$. In [3,12], it was shown that under the Penrose limit,

$$P_i\rightarrow e_i,\quad \begin{cases} M_{0i}, & i=1\ldots4\\ M_{9i}, & i=5\ldots8\end{cases}\rightarrow 2\mu e_i^*. \tag{7.1}$$

Therefore, our quotient by the $k_{S_{12}^+}$ isometry corresponds to a quotient by the linear combination P_1+M_{02}. Alternatively, we could map it to a discrete quotient of the sphere by P_5+M_{96}. Interestingly, Behrndt and Lüst have shown that $(AdS_5/\mathbb{Z}_N)\times S^5$ orbifolds are U-dual to $AdS_5\times(S^5/\mathbb{Z}_N)$ orbifolds [50]. We see that this obviously holds for $\mathbb{Z}$ orbifolds as well, at least after taking the Penrose limit. It would be interesting to understand if this is true before taking the Penrose limit, as well.

On the AdS side, this $\mathbb{Z}$ quotient is one of those discussed by Ghosh and Mukhi [23]. Specifically, introducing "light-cone" coordinates in the embedding space,

$$z_1^\pm=Y^0\pm Y^2,\quad z_2^\pm=Y^1\pm Y^{-1},\quad w=Y^3\pm iY^4, \tag{7.2}$$

and the corresponding AdS_5 coordinates

$$z_1^\pm=\cosh\frac{\theta_1}{2}e^{\pm\delta},\quad z_2^\pm=\sinh\frac{\theta_1}{2}\cos\frac{\theta_2}{2}e^{\pm\alpha},$$

$$w=\sinh\frac{\theta_1}{2}\sin\frac{\theta_2}{2}, \tag{7.3}$$

then the quotient is by $\partial/\partial\delta+\partial/\partial\alpha$. This group action is free, and preserves half the supersymmetries. In Sec. III, it was shown that after taking the Penrose limit, it preserves 3/4 of the supersymmetries. It is already known that enhancement of supersymmetry often occurs upon taking the Penrose limit [8,9,13,15,17,18]. Here we have the novel phenomenon that the supersymmetry is enhanced not to the full 32 supercharges but to 24.

The more complicated $k_{S_{15}^+}$ mixes the AdS with the sphere via P_1+M_{95}. The Melvin twist adds some M_{34} (or M_{78}).

VIII. CONCLUSIONS

The most general set of spacelike isometries on which one can compactify the pp wave has been identified. In contrast with Minkowski space, there are a large number of choices for compactification on supersymmetric circles and tori, even before considering the shape of the torus. Some of these are given in Table I. Even more remarkably, one finds compactifications to nine dimensions which preserve 20 or 24 supercharges, and compactifications to eight dimensions which preserve 18 or 20 supercharges. Whether these peculiar amounts of supersymmetry arise in a way related to the 3/4 BPS states of [20] bears investigating. T duality gives type IIA and 11-dimensional supergravity solutions that also preserve precisely 24 supercharges.

It should be noted that it is easy to see a very similar story for the maximally supersymmetric M-theory wave [44]. In particular, using the notation of [44], compactification on circles generated by $k_{e_1}+(3/\mu)k_{e_2^*}$ and $k_{e_4}+(6/\mu)k_{e_5^*}$ will preserve 24 supercharges.

Understanding the CFT dual to the compactifications described here would be very interesting. In Sec. VII, it was seen that the pp wave compactification arises from a free, and fairly simple, $\mathbb{Z}$ orbifold of AdS_5, of a type discussed in [23]. However, the simpleness of the orbifold could be quite deceptive; it has been noted in [51,52] that an orbifold of AdS by a discrete group is typically dual to a CFT on a non-Hausdorff space. For this reason, it may make more sense to try to consider, using the techniques of [53], the equivalent description of the pp wave compactification via an $S^5/\mathbb{Z}$. (Finite orbifolds have recently been discussed in this context in [13,15,17,18].)

This problem is unlikely to improve for the orbifolding that includes the Melvin twist. However the rewards of such an investigation could include a definitive resolution of the fate of the Melvin tachyon. (An interesting analysis of a stable twisted compactification of $AdS_7\times S^4$ was given in [54].)

Finally we saw that simultaneous compactification on S_{12}^+ and S_{21}^- results in a noncompact, nonabelian orbifold that preserves 24 supercharges, and includes a null identification. It would be interesting to study this orbifold.

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APPENDIX A: MORE MANIFEST ISOMETRIES

In the text compactification was considered only along $k_{S^+_{ij}}$, but it was noted that it is possible to consider the T^2 compactification along $k_{S^\pm_{12}}$, say. This is made manifest via the coordinate transformation

$$x^+ = y^+, \quad x^- = y^- + 2\mu y^1 y^2 \sin(4\mu y^+), \quad x^I = y^I, \tag{A1a}$$

$$x^1 = -(y^1+y^2)\cos(2\mu y^+),$$
$$x^2 = -(y^1-y^2)\sin(2\mu y^+). \tag{A1b}$$

The field configuration in these coordinates is

$$ds^2 = 2dy^+dy^- - 4\mu^2 y^I y^I (dy^+)^2 + (dy^1)^2 + (dy^2)^2 + 2\cos 4\mu y^+ dy^1 dy^2 + dy^I dy^I, \tag{A2a}$$

$${}^{(5)}F = \frac{1}{8} d\cos(4\mu y^+) dy^1 dy^2 dy^3 dy^4 + \mu dy^+ dy^5 dy^6 dy^7 dy^8. \tag{A2b}$$

Note that the metric—and the coordinate transformation—is singular at $y^+ = n\pi/4\mu$. Of course, these are just coordinate singularities. The analysis of Sec. III shows that this compactification preserves 16 supercharges.

APPENDIX B: KILLING SPINORS IN THE X^μ COORDINATE SYSTEM

In the coordinate system (2.3a), (2.3b) and using the zenbein (4.13), the Killing spinors are

$$\epsilon(\psi) = [\mathbb{1} + \mu X^1 \Gamma^{\check{+}\check{2}} - \mu X^2 \Gamma^{\check{+}\check{1}} - iX^i\Omega_i][\cos(\mu X^+)\mathbb{1} + \sin(\mu X^+)\Gamma^{\check{1}\check{2}}][\cos(\mu X^+)\mathbb{1} - i\sin(\mu X^+) I] \times [\cos(\mu X^+)\mathbb{1} - i\sin(\mu X^+) J]\psi. \tag{B1}$$

Note that Ω_μ has the same form (2.10) in this coordinate and frame. It was already noted in Sec. III, via Eq. (2.13), that the Killing vector $k_{S^+_{12}} = -\partial/\partial X^1$ preserves 24 supersymmetries. In this coordinate system the Lie derivative (2.13) with respect to $k_{S^+_{12}}$ is just

$$\pounds_{k_{S^+_{12}}} \epsilon(\psi) = -\frac{\partial}{\partial X^1}\epsilon(\psi) = \epsilon(i\mu\Gamma^{\check{2}}(\Gamma^{\check{3}\check{4}} - i\mathbb{1})\Gamma^{\check{+}}\psi), \tag{B2}$$

which (again) vanishes precisely for $(\Gamma^{\check{3}\check{4}} - i\mathbb{1})\Gamma^{\check{+}}\psi = 0$, so the 24 Killing spinors are independent of the X^1 coordinate.

Reduced to nine dimensions, the Killing spinor equation (2.12) is

$$\mathcal{D}_\mu \epsilon = \left[\nabla_\mu + \frac{i}{4}{}^{(2)}F_{\mu\nu}\Gamma^\nu + \frac{1}{6}{}^{(4)}F_{\mu\alpha\beta\gamma}\Gamma^{\alpha\beta\gamma} - \frac{1}{24}{}^{(4)}F_{\alpha\beta\gamma\delta}\Gamma^{\alpha\beta\gamma\delta}{}_\mu\right]\epsilon = 0. \tag{B3}$$

The integrability condition $[\mathcal{D}_+, \mathcal{D}_2]\epsilon = 0$ implies $(\Gamma^{\check{3}\check{4}} - i\mathbb{1})\Gamma^{\check{+}}\epsilon = 0$ which is another way of demonstrating that there are twenty-four supercharges in nine dimensions. The other components of the integrability condition vanish.

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